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## SIZE, SPRAWL, SPEED AND THE EFFICIENCY OF CITIES

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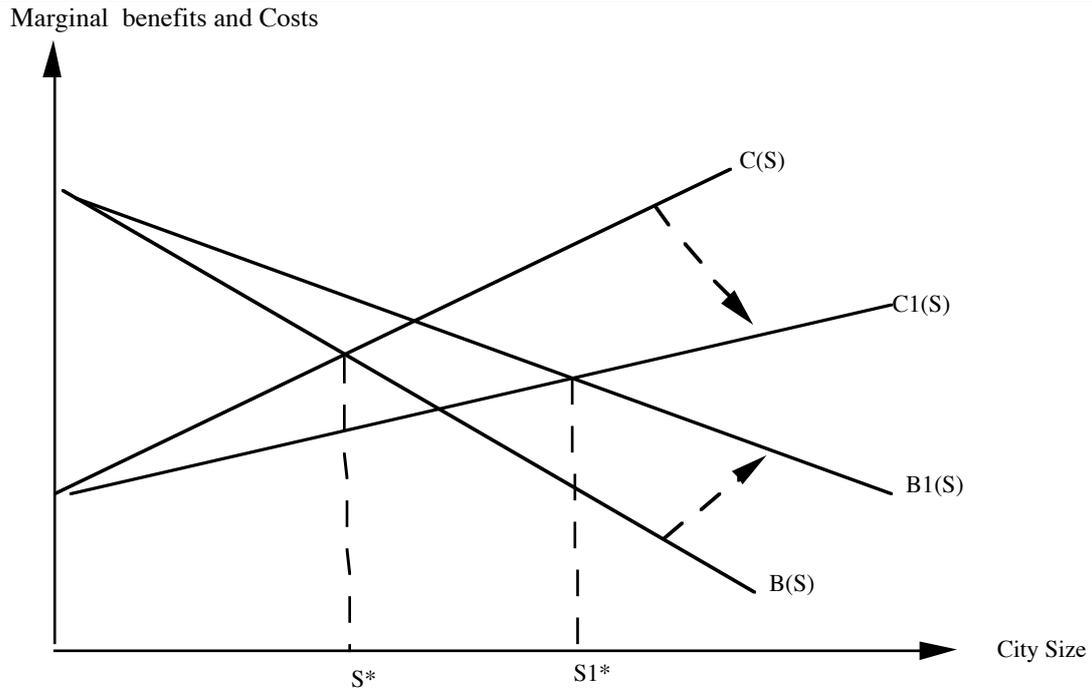
# Size, Sprawl, Speed and the Efficiency of Cities

## *Introduction*

Some cities are more efficient than other. Why? At a time when most people live and most activities take place in cities, identifying the determinants of the efficiency of cities is of importance not only for city planners, but for also for macro-economists. This paper considers three potential determinants of urban efficiency, the three « S »: the *size* of the city, the *speed* at which people and goods are moved in the city, and the *sprawl* or the relative location of jobs and homes in the city. It tries to measure the relative contribution of these three determinants. In this paper, efficiency is in general defined as labor productivity, that is output per worker. Total productivity would be a better indicator of efficiency, but data on total productivity of cities is difficult, not to say impossible, to obtain; in addition a study on the « surproductivity » of Paris relative to the rest of France (the ratio of Paris productivity to the productivity of the rest of France), in which total productivity was estimated, showed that labor surproductivity was a very good estimator of total surproductivity (Rousseau 1995).

The relationship between urban productivity and urban size has been recognized and studied for a long time. In a seminal contribution, Alonso (1971) developed a model that assumed that both benefits and costs increased with city size, with the benefit curve increasing less and less and the cost curve increasing more and more. It follows that there is a city size for which the difference between benefits and costs, also called the net benefit, is maximal, and which is the so-called optimal size of cities. In marginal terms, there is a downward sloping marginal benefit curve  $B(S)$  and an upward sloping marginal cost curve  $C(S)$ : the point at which they intersect defines the optimal size  $S^*$  of cities, as represented in Figure 1.

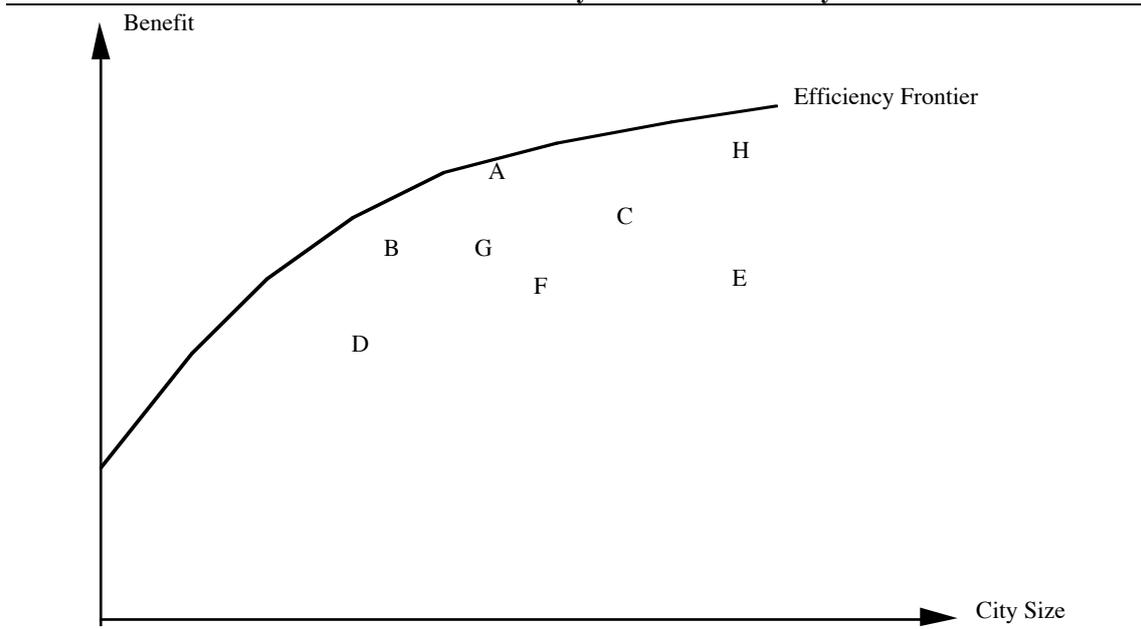
**Figure 1 - Optimal City Size as a Function of City Management**



This simple and elegant model had the great advantage of drawing attention to the obvious, but often forgotten, fact that larger cities have benefits as well as costs. But it ignored a key dimension: the management of cities, that modifies the benefit and cost curves. Not all cities are equally well managed. Tokyo, the largest world city, is probably not too large, because it is reasonably well managed. There are, in some parts of the world, cities of 200,000 people which are definitely too large, because they are very poorly managed. Good management can —and should— lower the marginal cost curve and turn it into  $C_1(S)$  and raise the marginal benefit cost curve to  $B_1(S)$ . The intersection of these curves defines a new optimal size  $S_1^*$  much to the right of  $S^*$ . Good management can therefore increase indefinitely the « optimal » size of a city.

Another way to tell the same story is to consider that the benefits associated with city size are only potential, that they are contingent upon the quality of management. City size would therefore define an efficiency frontier, with effective efficiency often significantly below this frontier, as suggested in Figure 2. The distance between a particular point (that is a city) and the frontier is a measure of the quality of its management.

**Table 2 — Urban Efficiency as a Function of City Size**



One way to give some flesh to these rather abstract constructs is to identify mechanisms by which « management » can influence productivity. The hypothesis put forward here —and tested— is that the efficiency of the transport system (in short: speed) and the relative location of jobs and homes (in short: sprawl), which are the output of transport policies and urban policies respectively, combine with city size to determine the effective size of the labor market. This effective size of the labor market —the number of jobs that can, on average, be reached in less than t minutes— is in turn is a major explanation of labor productivity.

The first of these relationships is rather self evident. The closer people are from the jobs, all other things equal, the larger the effective labor market; similarly the higher the speed at which people go to their jobs, the larger the effective labor market; and the larger the city size, all other things constant, the larger the effective labor market.

The second relationship is also easy to understand. A larger effective labor market makes it easier for enterprises to find the skills they need, and for workers to find the jobs they want. What counts therefore is not only the size of the city, but the effective labor market size. In a small city, the effective labor market at let us say 40 minutes is about equal to the number of jobs/workers. Every worker can access all the jobs in less than 40 minutes. Not so in a large city. Many workers cannot access many of the city jobs in less than 60 minutes, and the effective size of the labor market is only a fraction of the total number of jobs/workers, a fraction that varies with transport and land use patterns, that is with urban management.

*Measuring the effective labor market size*

Let us consider an agglomeration divided in n zones labelled 1, 2, .. i, .. j, ... n. The larger the number of zones, the better. We have:

$W_i$  = the number of workers located in zone i, with  $\sum_i W_i = W$ ;

$J_i$  = the number of jobs located in zone  $i$ , with  $\sum_i J_i = J$ ;

$T_{ij}$  = the time it takes to go from zone  $i$  to zone  $j$ ;

The effective size of the labor market at  $t$  minutes can be defined either from the viewpoint of workers or from the viewpoint of enterprises:

$L(t)$  = the effective size of the labor market for workers (labourers);

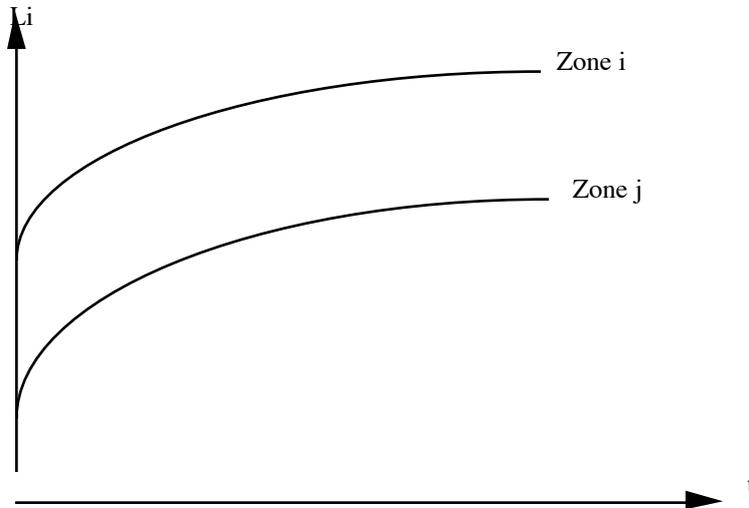
$E(t)$  = the effective size of the labor market for enterprises;

$L_i(t)$  = the effective size of the labor market for the workers of zone  $i$ .

For a given zone  $i$ , we have:

$$L_i(t) = \sum_j J_j \text{ for } j \text{ such that } T_{ij} < t$$

**Figure 3 — Effective Size of the Labor Market ( $L_i(t)$ ) as a Function of Time ( $t$ ) and Zone ( $i$ )**



For a given agglomeration,  $L_i(t)$  is a function of  $t$ , and varies with each zone  $i$ . To take an example, for a zone located in downtown Paris,  $L_{\text{center}}(60)$  is equal to about 4 million jobs and  $L_{\text{center}}(45)$  to 2.7 million, whereas for a zone located in the periphery at about 30km of the center,  $L_{\text{periph}}(60)$  is about 2.9 million and  $L_{\text{periph}}(45)$  only 1.2 million. This is illustrated in Figure 3, in which zone  $i$  is a centrally located zone whereas zone  $j$  is a peripherally located zone.

For the entire agglomeration, the effective size of the labor market is the weighted average of the labor market size of all zones, weighted by the relative number of workers in each zone:

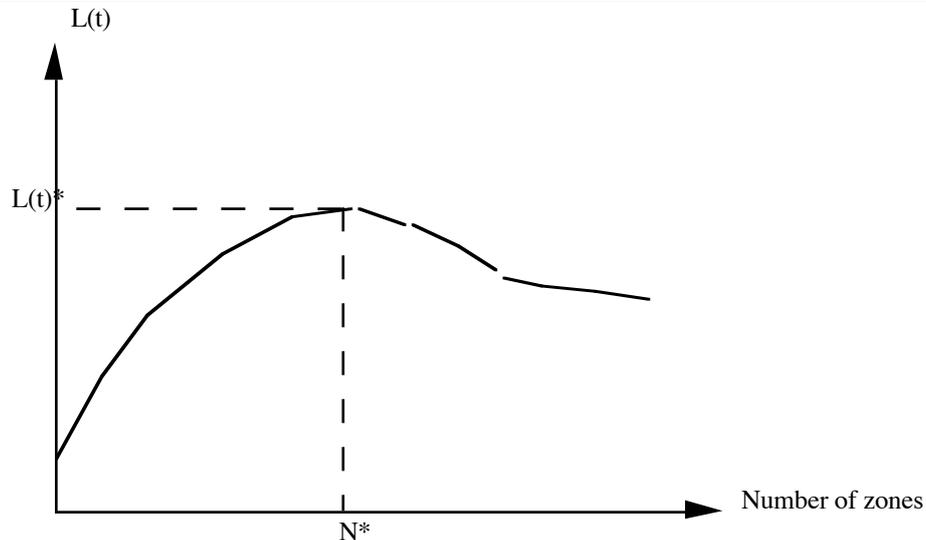
$$L(t) = \sum_i L_i(t) * W_i / W$$

$$L(t) = \sum_i \sum_j J_j * W_i / W \text{ for } j \text{ such that } T_{ij} < t$$

For a given travel time  $t$ , the effective size of the labour market is a function of the area considered or of the number of zones taken into consideration. However, it is

interesting to note that this function has a maximum. When only a few zones are considered, the labor market size is bound to be small. As the number of zones increases, so does the labor market size. But there comes a time when increasing the area no longer increases the labor market size. This is because the labor market size of each of the peripheral zones which are added tends to be small, and to push the average down. One can therefore define the geographical size  $N^*$  of the agglomeration for which the effective labor market size is maximized and equal to  $L^*$ . This is represented in Figure 4.

**Figure 4 — Effective Size of the Labor market as a Function of Area Size**



To again take the example of Paris, for the entire agglomeration  $L(60)$ , the effective size of the labour market at 60 minutes, is about 2,7 million jobs, whereas  $L(45)$  is about 1.2 million jobs. The largest effective size of the market at 60 minutes  $L(60)^*$  is 3.1 million and for 45 minutes  $L(45)^*$  it is 2 million.

Similar concepts, measures, and relations can be developed for  $E(t)$ , the effective size of the labour market from the viewpoint of enterprises:

$$E_i(t) = \sum_j W_j \text{ with } j \text{ such that } T_{ij} < t$$

$$E(t) = \sum_i E_i(t) * J_i / J$$

*Relationship between productivity and labor market size*

The hypothesis that the productivity of a city is a function of the effective labor market size of the city is supported by the few case studies that we have conducted on this topic. A first study compares three Korean cities: Seoul, Busan and Daegu. Table 1 presents the relevant data.

**Table 1 - Productivity and Effective Size of the Labor Market, Three Korean Cities, circa 1990**

	Population (in 1000)	Employment (in 1000)	L(60) (in 1000)	E(60) (in 1000)	Productivity (in 1000 won)
Seoul (1987)	16,792	5,697	2,911	3,165	13,984
Busan (1994)	4,187	1,762	1,361	1,352	10,588
Daegu (1987)	2,107	807	754	755	9,932

Notes: L(60)=Effective size of the labor market at 60 minutes from the viewpoint of workers;  
E(60)=Effective size of the labor market at 60 minutes from the viewpoint of enterprises; productivity numbers are for the same year 1992. Productivity is the output per worker.

The first two columns are mostly for reference. A comparison of the second with the third and fourth columns shows that in large cities, the effective size of a labor market is very different from the total number of jobs in the city. In Seoul, the average worker has in 60 minutes access to only 51% of all the jobs offered by the city; and the average enterprise has 56% of all the workers at less than 60 minutes. In a smaller city like Daegu, these percentages are much higher: 93%. What matters here for our purpose is the relationship between the last column, productivity (output per worker), and the two previous columns, effective size of the labor market. This relationship appears to be significant. We have:

$$\ln \text{ Productivity} = 7.5 + 0.24 * \ln L(60) \quad R^2=0.97$$

(17.2)      (4.1)

Three points are not much to run a regression, and the coefficient 0.24, the elasticity of productivity with respect to L(60), the effective size of the labor market from the viewpoint of workers, must be taken with care. It suggests that a 10% increase in the labor market size is accompanied by a 2.4% increase in productivity and therefore in output.

A second study compares 22 French cities, excluding Paris, for which transport surveys were available, making it possible to calculate effective labor market sizes. The city productivity index utilized takes into account differences in the activity mix, by means of a sort of shift-share analysis, so as to retain « pure » estimates of output, and hence of productivities. To be more rigorous, let:

$Y_k$  = output of city k;

$L_{s,k}$  = Labor force in sector s in city k;

$L_s$  = Labor force in sector s in France ( $L_s = \sum_k L_{s,k}$ );

$Y_s$  = Output of sector s in France;

$p_s = Y_s / L_s$  = productivity of sector s in France;

$Y_k^*$  = Implicit output of city k ( $Y_k^* = \sum_s L_{s,k} * p_s$ );

$p_k$  = pure or relative or adjusted productivity of city k ( $p_k = Y_k / \sum_s L_{s,k} * p_s$ )

The pure or relative productivity of a city  $k$  is equal to its actual output  $Y_k$  divided by its implicit output  $Y_k^*$ . The implicit output is defined as the output that would prevail in the city if labor productivity in each of the sectors of that city were that prevailing in the country as a whole. Take a city  $k$ . The structure of the labor force in that city is known. So is the output per worker in each of the sectors at the national level. One can figure out what the city output would be if the city productivity in each of the sectors was that prevailing at the national level. Let us assume it would be 100 mF. It turns out to be 120 mF, because city  $k$  is more eproductive than the average. Its relative labor productivity is then 1.2.

Table 2 presents the relationships established between productivity and labor market size.

**Table 2 — Elasticities of Productivity With Respect to Labor Market Size, 22 French Cities, Circa 1990**

Type of Labor Market	Elasticity	T values	Intercept	R2
From the viewpoint of workers				
At 20 min (L(20))	0.24	5.1	9.17	0.56
At 25 min (L(25))	0.18	4.5	9.76	0.50
At 30 min (L(30))	0.15	4.1	10.1	0.46
From the view point of enterprises				
At 20 min (E(20))	0.18	4.2	9.9	0.46
At 25 min (E(25))	0.15	4.1	10.1	0.46
At 30 min (E(30))	0.13	3.9	10.6	0.43

Note: Elasticity is the value of  $b$  in:  $\ln \text{productivity} = a + b \cdot \ln \text{labor market size}$

The relationship seems quite robust. A larger size of the effective labor market size results in a higher productivity. The elasticities are greater for 20 minutes labor markets than for 25 or 30 minutes labor market. They are also greater for the labor market from the view point of workers. These elasticities vary from 0.13 to 0.24. An elasticity of 0.18 seems a reasonable order of magnitude. When the labor market size increases by 10%, productiviyy — and therefore output — increases by slightly less than 2%.

### *Measuring sprawl (D)*

Our hypothesis is that the effective size of the labor market is a function of the geography of the area, that is the relative location of jobs and homes, in short its sprawl, and of the efficiency of the transport sector, the speed at which trips are made. These two concepts must be defined more precisely.

Sprawl is defined here as the average potential job-home distance. Let:

$L_i$ ,  $W_i$ ,  $L$ , and  $W$  be defined as above;

$d_{ij}$  = the Cartesian distance between zone  $i$  and zone  $j$ , in km;

$DE_i$  = the potential job-home distance for the enterprises of zone  $i$ , in km;

$DL_i$  = the potential home-job distance for the workers of zone  $i$ , in km;

D = the potential job-home (or home-job) distance for the agglomeration, in km.

We have:

$$DE_i = \sum_j d_{ij} * L_j / L$$

which is the average distance to workers for the enterprises of zone i weighted by the number of potential workers in each zone j.  $DE_i$  varies with the zone considered, and it is an increasing function of the distance of zone i to the center. In the case of Paris, for instance, it is 7.3 km for the central zones, and over 70 km for the peripheral zones.

We then have:

$$D = \sum_i DE_i * W_i / W$$

$$D = \sum_i \sum_j d_{ij} * L_j * W_i / L * W$$

$DL_i$  can be defined in the same way, and it can be shown that

$$\sum_i DL_i = \sum_i DE_i = D$$

In the sample of French cities for which this indicator was calculated, the average potential job-home distance is 6.4 km. It is of course larger than the average effective job-home distance, which is 3.3 km, because people do not take jobs randomly but chose jobs closer to their home, all other things equal. There are great variations between cities, even between cities of a similar population size. D varies from 3.3 km in Amiens to 11 km in Lille. Valenciennes and Grenoble have about similar population sizes (around 340,000 inh.), but D is 9.7 in Valenciennes and only 5.0 in Grenoble. In the case of Paris, D is equal to 23.0 km and the effective job-home distance to 9.8 km.

Other indicators of "sprawl", such as density gradients of population or employment, could be defined and utilized. We preferred our "potential job-home" concept, however, for two reasons. One is that it does not imply any hypothesis on the shape of the city. The other is that it lends itself more readily to the analysis undertaken.

### *Measuring Speed (V)*

The efficiency of the transport system in a city is defined as the average speed at which people go from origin to destination in that city. Transport surveys record the time  $T_{ij}$  it takes to go from zone i to zone j. The distance  $d_{ij}$  between zone i and zone j is the Cartesian distance, as the crow flies. Not all pairs of (i,j) are documented, but a sufficiently large number is, since the number of households surveyed is usually in the thousands, and reaches 20,000 in the case of Paris. Speed V is therefore:

$$V = \sum_{ij} d_{ij} / \sum_{ij} T_{ij}$$

The speed thus defined is not the speed at which people actually drive their cars or ride buses, for two reasons. It takes into account the availability of roads and bridges as well as the topography of the area. A new bridge over a river, that shortens the actual distance travelled, even if actual speeds in the city are not modified, will increase the

value of V. Then, the time  $T_{ij}$  used is the total time it takes to go from origin to destination, including access time to and from a car or to and from a transit terminal.

Speed thus defined was calculated for our sample of 22 French cities, and for Paris, by mode (car, transit), by areas (downtown, rest of the area) and by trip purpose (work, non work). The main findings are presented in Table 3.

**Table 3 — Transport Speeds in France, circa 1990**

	Paris (km/hour)	23 French cities (km/hour)
All trips	13.9	13.2
Private cars trips	16.3	15.3
Transit trips	11.6	8.5
Downtown trips	6.5	6.3
Journey to work trips		14.4
Other purposes trips		12.7
Peak-hour trips		13.1
Off-peak hour trips		13.2

Note: Speed is defined as distance as the crow flies divided by total time of transportation, including access time.

The average speed appears to be 13.2 km/hour for the 22 French cities, and slightly faster for Paris (13.9). Speed is nearly twice as high for private car transportation (15.3) than for transit (8.46), particularly in the case of Paris. It is much smaller for downtown transport (6.3) than for the entire agglomeration (13.2). Surprisingly enough, speed is higher for journey-to-work trips than for other purposes trips, and also for peak-hour trips. The explanation of this apparent paradox is twofold. First, the share of the faster mode (the car) is larger in journey-to-work trips. Second, journey-to-work trips are longer, and as a consequence faster, than trips undertaken for other purposes. The same observations apply to the comparison of peak and off-peak trips, which are carried out at about the same speed.

Average speed varies from city to city, and as a function of city size. Except for the three largest French cities (Lyons, Marseilles and Lille), speed increases with the size of the agglomeration, probably because longer, and faster, trips weight more in larger agglomerations.

#### *Explaining the Effective Size of the Labor Market*

For an agglomeration of a given size (S), the effective size of the labour market (E or L) will be negatively affected by sprawl (D) and positively affected by speed (V):

$$E \text{ (or L)} = f(S, D, V)$$

Table 4 presents the coefficients of the regression analysis conducted for L(25) and E(25).

**Table 4 — Coefficients of Regression Analysis Explaining Efficiency by Size, Sprawl and Speed, 22 French cities, Circa 1990**

	Dependent Variable	Intercept	Size (S)	Sprawl (D)	Speed (V)	R2	Form
(1)	L(25)	-91.0 (-2.9)	0.202 (9.3)	-16.87 (-4.32)	16.04 (4.67)	0.89	Linear
(2)	E(25)	-42.5 (-1.31)	0.183 (8.22)	-15.00 (-3.73)	12.36 (3.46)	0.86	Linear
(3)	L(25)	-4.29 (-2.29)	1.07 (8.30)	-1.17 (-3.75)	1.79	0.88	Log-Log
(4)	E(25)	-2.86 (-2.29)	0.97 (8.27)	-1.12 (-3.93)	1.46 (2.90)	0.87	Log-Log

Notes: L(25) is the effective labor size of the labor market at 25 minutes from the view point of workers; R(25) is the same concept from the view point of enterprises; Size is the population of the agglomerations, in 1,000; Sprawl is the average potential job-home distance; speed is the average speed as defined in the text; number in parentheses are the T values.

The model explains fairly well the labor market size, both in its linear form and in its exponential form. R2 are high, all explanatory variables are highly significant and of the expected signs. Four points stand out.

The elasticities of labor market size to population is close to 1. This is to be expected. When the size of a city increases by 10%, the effective size of the labor market also increases by about 10%. The 0.20 or 0.18 coefficients of the linear regressions (1) and (2) can be interpreted as activity ratios. When the city size increases by 100 persons, the labor market increases by about 20 jobs and 18 workers within 25 minutes.

The elasticities of labor market size with respect to sprawl are -1.12 and -1.17. When the average potential job-home distance increases by 10%, the effective size of the labor market decreases by about 11.5%. What equations (1) and (2) tells us is that when the job-home distance increases by one km, the size of the labor market at 25 minutes is reduced by about 16,000 jobs, all other things equal.

The elasticities of labor market size with respect to average transport speed are 1.46 and 1.79. This means that a 10% increase in average speed, all other things constant, leads to a 15-18% increase in the labor market size.

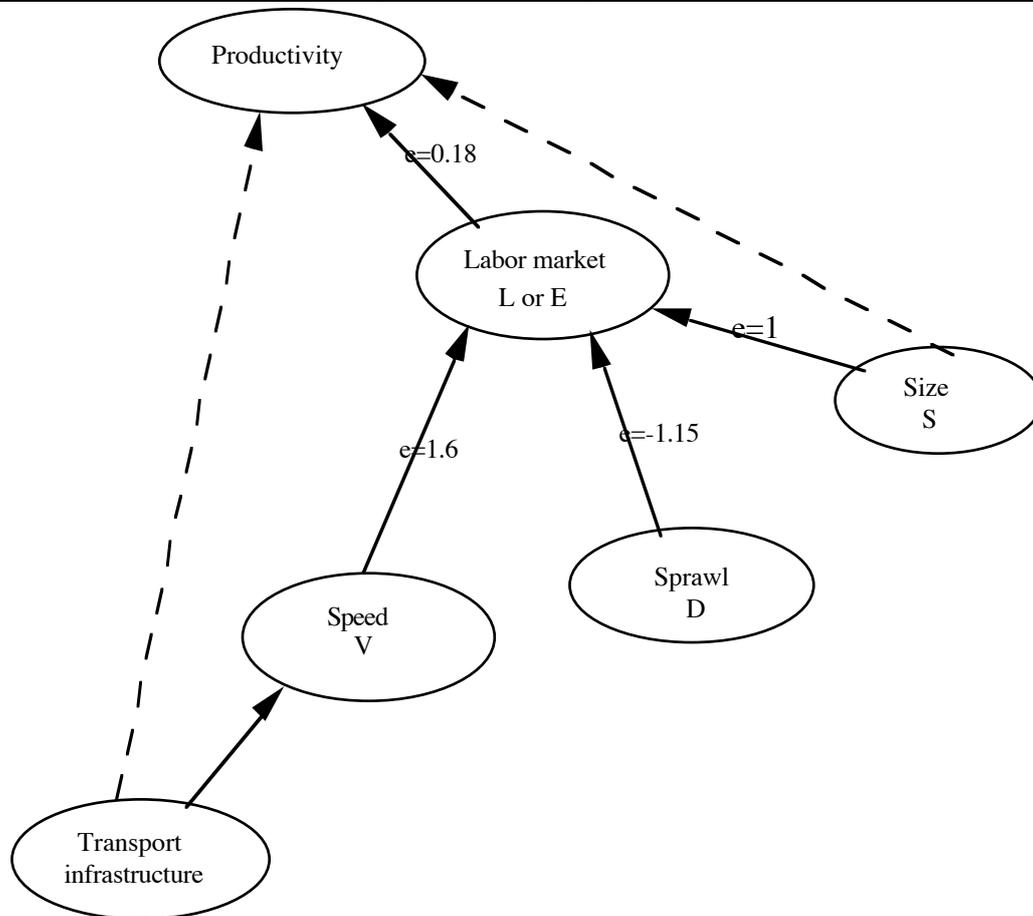
It also appears that the labor market from the view point of workers is more elastic to size, sprawl or speed than the labor market from the view point of enterprises. This is probably because workers homes are more dispersed than enterprises. It means that workers tend to gain more than enterprises when a city grows, when sprawl is contained and when transport improvements are made.

Regressions not reported in Table 4 also suggest that elasticities are more important when the labor market is defined at 20 minutes than when it is defined at 25 minutes or at 30 minutes.

## Conclusion

The —admittedly limited— data supports the theory that the efficiency of a city is a function of the effective size of its labor market, and that this labor market size is itself a function of the overall size of the city, but also of its sprawl and of the speed at which trips in the city are made. Elasticities reflecting these relationships have been produced. They are shown in Figure 5.

Figure 5 — Efficiency of Cities



The findings of the econometric analysis conducted on 22 French cities are very much in line with the non econometric conclusions of a comparison of Paris and London (CEBR & OEIL, 1997) with which the authors were associated. Such a comparison is delicate because it is relatively easy to tell where the Paris agglomeration ends, but difficult to find out where the London agglomeration ends. The study showed that, by and large, Paris is more productive than London, or more precisely that the ratio of Paris productivity to France productivity is greater than the ratio of London productivity to the UK productivity. This is associated with, and according to us explained by, a much larger effective labor market size in Paris. This in turn is explained by the fact that London is at the same time more widespread and less transport-efficient than Paris. Our indicator for sprawl (or any indicator of sprawl for that matter) is much larger for London than for Paris. Transport speed is greater in Paris than in London. The latter is explained by the marked difference in transport

infrastructure spending patterns over the past decades: Paris has invested much more than London, in public transportation and even more so perhaps in roads.

Let us assume that speed, the efficiency of the transport system, is, at least in part, explained by the transportation infrastructure endowment of cities. The dotted lines of Figure 5 refer to the fairly traditional analysis relating infrastructure endowment and productivity, for one, or city size and productivity, for the other. These traditional relationships, which are strong and well established, have been criticized for their « black box » nature. What we have done has been to lift the top of the black box, and to have a look at what is inside.

We found that, as could easily be expected, urban policies and transport policies play a significant and measurable role in determining the efficiency of an agglomeration. Containing sprawl, and improving transportation speed in a city both increase the productivity and therefore the output of the city. One can even estimate the magnitude of this contribution. Increasing speed in a city by 10% increases productivity by 2.9%. This finding is established on a cross-section analysis. Its extrapolation for time-series analysis is not fully warranted. But it is established all other things constant and what happens over space probably gives us an idea of what happens over time. Assuming this, if we could know by how much a given transport investment increases transport speed, we could use this relationship to estimate the rate of return of the investment.

This can be attempted on the case of Paris. In another paper (Prud'homme 1998), we estimated that transport investments undertaken in the Paris area over the 1983-91 period, for an (after tax) amount of 45 billion francs, increased traffic speed by about 5%, relative to what would have happened in the absence of such investments. If we use this 0.29 elasticity, this means that productivity and output in Paris was increased by about 1.44% as a result of transport investments. This represents an increase in output of about 29 billion francs, which would translate into a 64% immediate rate of return<sup>1</sup>. This is a very high rate indeed, although one which is in line with some of the estimates produced by cost-benefit analysis of transport projects in the Paris region and also by some of the estimates produced by production function analysis. It could also be that the elasticities calculated on the basis of 22 French cities (excluding Paris) cannot easily be extrapolated to the case of Paris, which is much larger than these cities.

This analysis could be refined and expanded. More such studies would be required to find out whether the elasticities put forward are sufficiently robust. One could also take into account the size of « activity market », in addition to the labor market, defined as the number of jobs which can be accessed in less than t minutes from the viewpoint of enterprises, to reflect the fact that the ease with which enterprises deal with each other can contribute to productivity. This activity market size would be explained by the efficiency of the transport system (the speed) and by the job-to-job potential distance. One could also differentiate between types of jobs, and identify, then study, different

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<sup>1</sup> The immediate rate of return of an investment I producing a yearly benefit B during the first year is defined as B/I. Under reasonable assumptions (about the rate at which B increases over time), the immediate rate of return can be shown not to be very different from the standard internal rate of return, the rate that equalizes the discounted flows of investments and of benefits.

labor markets and their sizes. One could also replace the rather crude concept of a labor market at  $t$  minutes by more sophisticated indicators of accessibility. Finally, one could, and should, explore the relationship that exists between speed and sprawl.

## References

Alonso, W. 1971 "The Economics of Urban Size." *Papers of the Regional Science Association*, 26 pp. 71-83.

CEBR & OEIL. 1997. (Centre for Economics and Business Research Ltd & Observatoire de l'Economie et des Institutions Locales, for the corporation of London). *Two Great Cities: A Comparison of the Economies of London and Paris*. London. The Corporation of London. 181p.

Rousseau, Marie-Paule. 1995. *Les avantages de la concentration urbaine: une approche par la mesure de la productivité des grandes villes*. PhD dissertation, Univ. of Paris XII

Prud'homme, Rémy. 1996. « Megacities Management; Institutional Dimensions » in; Stubbs, J & G. Clarke (ed.) *Megacities Mananagement in the Asia and Pacific Region*. vol 1, pp. 99-129

Prud'homme, Rémy. 1998. « Estimating the Economic contribution of large transport investments: the Case of the Paris Region », paper prepared for a book *entitled Making Decisions for Mega Projects*, to be edited by Harry Dimitriou and Bent Flyvberg